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On sliding mode observers for systems with unknown inputs†

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Abstract

This paper considers the problem of designing an observer for a linear system subject to unknown inputs. This problem has been extensively studied in the literature with respect to both linear and nonlinear (sliding mode) observers. Necessary and sufficient conditions to enable a linear unknown input observer to be designed have been established for many years. One way to express these conditions is that the transfer function matrix between the unknown input and the measured output must be minimum phase and relative degree one. Identical conditions must be met in order to design a 'classical' sliding mode observer for the same problem. This paper shows how the relative degree condition can be weakened if a classical sliding mode observer is combined with sliding mode exact differentiators to essentially generate additional independent output signals from the available measurements. A practical example dedicated to actuator fault detection and identification of a winding machine demonstrates the efficacy of the approach.

keywords: Unknown input observers; sliding mode; sliding mode differentiators; actuator faults

1 Introduction

The problem of designing an observer for a multivariable linear system partially driven by unknown inputs is of great interest. Such a problem arises in systems subject to disturbances or with inaccessible/unmeasurable inputs and in many applications such as fault detection and isolation, parameter identification and cryptography. This problem has been studied extensively for the last

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two decades. One approach has been to design linear observers (both full-order and reduced-order). In the literature, linear observers which are completely independent of the unmeasurable disturbances are known as Unknown Input Observers (UIOs) [3, 4, 5, 6, 22]. In particular, easily verifiable system theoretic conditions, which are necessary and sufficient for the existence of UIOs, have been established (see for example [20] or [22]). One possible statement of these conditions is that the transfer function matrix between the unmeasurable input and the measured outputs must be minimum phase and relative degree one.

The concept of sliding mode control [12, 27, 33] has been extended to the problem of state estimation by an observer, for linear systems [33], uncertain linear systems [11, 35] and nonlinear systems [1, 9, 29]. Using the same design principles as for variable structure control, the observer trajectories are constrained to evolve after a finite time on a suitable sliding manifold by the use of a discontinuous output injection signal (the sliding manifold is usually given by the difference between the observer and the system output). Subsequently the sliding motion provides an estimate (asymptotically or in finite time) of the system states. Sliding mode observers have been shown to be efficient in many applications, such as in robotics [2, 21], electrical engineering [7, 16, 34], and fault detection [15, 17]. Of interest here is the fact that the formulation posed in [11, 35] can be viewed as an unknown input observer problem [14]. Consequently it is not surprising that the necessary and sufficient conditions for the existence of a 'classical' sliding mode observer[1] as described in [11, 35] is that the transfer function matrix between the unmeasurable inputs (or disturbances) and the measured outputs must be minimum phase and relative degree one. This paper attempts to broaden the class of systems for which these observers can be designed. Specifically, the paper shows how the relative degree condition can be weakened if a classical sliding mode observer is combined with sliding mode exact differentiators to generate additional independent output signals from the available measurements.

The structure of the paper is as follows: §2 discusses existing results and presents a lemma concerning the invariant zeros of a system which is vital to the scheme which is proposed in this paper. It also introduces an augmented output distribution matrix which is important for the subsequent developments. §3 discusses two types of observer: a so-called step-by-step observer incorporating the super-twisting algorithm which is used to estimate a sufficient number of output derivatives in finite time; using these 'additional' outputs a classical first order observer is described which estimates the system states and the unknown inputs. §4 describes the winding machine example to demonstrate the efficacy of the approach. Finally §5 makes some concluding remarks.

The notation used throughout is standard: $\mathbb{R}$ denotes the field of real numbers; $\mathbb{N}^*$ represents the set of positive integers and $\|\cdot\|$ represents the Euclidean norm for vectors and the induced spectral norm for matrices.

[1] A precise observer description will be given later in the paper.

2 Motivation and problem statement

This paper is concerned with the design of a sliding mode observer for a linear time-invariant system subject to unknown inputs or disturbances:

$$\dot{x} = Ax + Bu + Dw \tag{1}$$

$$y = \begin{bmatrix} y_1 & \cdots & y_p \end{bmatrix}^T = Cx, \ y_i = C_i x \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the output vector, $u \in \mathbb{R}^q$ represents the known inputs and $w \in \mathbb{R}^m$ stands for the bounded, unknown inputs. It is assumed that A, B, C and D are known constant matrices of appropriate dimension. It is further supposed that $m \leq p$. Without loss of generality, it can be assumed that rank $(C) = p$ and that rank $(D) = m$.

Consider a sliding mode observer of the form

$$\dot{\hat{x}} = A\hat{x} + Bu + G_l(y - C\hat{x}) + G_n v_c \tag{3}$$

where G_l and G_n are design gains and v_c is an injection signal which depends on the output estimation error in such a way that a sliding motion in the state estimation error space is induced in finite time. The objective is to ensure the state estimation error $e = x - \hat{x}$ is asymptotically stable and independent of the unknown signal w during the sliding motion.

As argued in [12] necessary and sufficient conditions to solve this problem are: the invariant zeros of $\{A, D, C\}$ lie in $\mathbb{C}_-$ and

$$\text{rank}(CD) = \text{rank}(D) = m. \tag{4}$$

Condition (4) is sometimes called the *observer matching condition*, and is the analogue of the well-known *matching condition* [10] for a sliding mode controller to be insensitive to matched perturbations. Then, as argued in [12], there exists a linear change of coordinates that puts the original system into the canonical form given by:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}y + B_1 u, \\ \dot{y} &= A_{21}x_1 + A_{22}y + B_2 u + D_1 w(t) \end{aligned} \tag{5}$$

Remark: These observers have also been recently used in the field of fault detection and identification [13, 32] where the unknown input w (in this case a fault) is reconstructed by analyzing the so-called *equivalent output error injection* (which is the counterpart of the equivalent control in the design of sliding mode controllers). Thus the observer in (3) can provide both estimation of the states and the unknown input signal.

Here, the aim is to extend the existing results so that a sliding mode observer can be designed for the system (1-2) when the standard matching condition (4) is not satisfied, i.e. when rank$(CD) < m$. To this end, introduce the notion of *relative degree* $\mu_j \in \mathbb{N}^*$, $1 \leq j \leq p$ of the system with respect to the output y_j, that is to say the number of times the output y_j must be differentiated in order to have the unknown input w explicitly appear. Thus, μ_j is defined as follows:

$$\begin{aligned} C_j A^k D &= 0, \text{ for all } k < \mu_j - 1 \\ C_j A^{\mu_j - 1} D &\neq 0. \end{aligned}$$

Without loss of generality, it is assumed that $\mu_1 \leq ... \leq \mu_p$.
The following assumptions are made:

- the invariant zeros of $\{A, D, C\}$ lie in $\mathbb{C}_-$
- there exists a full rank matrix

$$C_a = \begin{bmatrix} C_1 \\ \vdots \\ C_1 A^{\mu_{\alpha_1} - 1} \\ \vdots \\ C_p \\ \vdots \\ C_p A^{\mu_{\alpha_p} - 1} \end{bmatrix} \tag{6}$$

where the integers $1 \leq \mu_{\alpha_i} \leq \mu_i$ are such that $\text{rank}(C_a D) = \text{rank}(D)$ and the μ_{α_i} are chosen such that $\sum_{i=1}^{p} \mu_{\alpha_i}$ is minimal.

Before describing the proposed observer scheme, the following lemma will demonstrate that the invariant zeros of the triple $\{A, D, C\}$ and the newly created triple with additional (derivative) outputs $\{A, D, C_a\}$ are identical. Consequently, if the original system is minimum phase the new triple $\{A, D, C_a\}$ is both minimum phase and relative degree one and hence a 'classical' observer of the form given in (3) can be designed for $\{A, D, C_a\}$. This is the main idea of the paper.

Lemma: The invariant zeros of the triples $\{A, D, C\}$ and $\{A, D, C_a\}$ are identical.

Proof: Suppose $s_0 \in \mathbb{C}$ is an invariant zero of $\{A, D, C_a\}$. Consequently $\tilde{P}(s)|_{s=s_0}$ loses normal rank, where $\tilde{P}(s)$ is Rosenbrock's system matrix defined by

$$\tilde{P}(s) := \begin{bmatrix} sI - A & D \\ C_a & 0 \end{bmatrix}$$

Since by assumption $p \geq m$, this implies $\tilde{P}(s)$ loses column rank and therefore there exist non-zero vectors η_1 and η_2 such that

$$\begin{aligned} (s_0 I - A)\eta_1 + D\eta_2 &= 0 \\ C_a \eta_1 &= 0 \end{aligned}$$

From the definition of C_a, $C_a\eta_1 = 0 \Rightarrow C\eta_1 = 0$. Consequently

$$\begin{aligned}(s_0 I - A)\eta_1 + D\eta_2 &= 0 \\ C\eta_1 &= 0\end{aligned}$$

and so $P(s)|_{s=s_0}$ loses column rank where

$$P(s) := \begin{bmatrix} sI - A & D \\ C & 0 \end{bmatrix}$$

is Rosenbrock's System Matrix for the triple $\{A, D, C\}$. Therefore any invariant zero of $\{A, D, C_a\}$ is an invariant zero of $\{A, D, C\}$.

Now suppose $s_0 \in \mathbb{C}$ is an invariant zero of $\{A, D, C\}$. This implies the existence of non-zero vectors η_1 and η_2 such that

$$(s_0 I - A)\eta_1 + D\eta_2 = 0 \tag{7}$$

$$C\eta_1 = 0 \tag{8}$$

The first (sub) equation of (8) implies $C_1\eta_1 = 0$. Suppose $\mu_{\alpha_1} > 1$. Then multiplying (7) by C_1 gives

$$s_0 \underbrace{C_1\eta_1}_{=0} - C_1 A\eta_1 + \underbrace{C_1 D}_{=0} \eta_2 = 0$$

which implies $C_1 A\eta_1 = 0$. By an inductive argument it follows that $C_1 A^k \eta_1 = 0$ for $k \leq \mu_{\alpha_1} - 1$. Repeating this analysis for C_2 up to C_p it follows

$$C_j A^k \eta_1 = 0 \qquad \text{for } k \leq \mu_{\alpha_j} - 1, \quad j = 1 \dots p$$

and therefore

$$C_a\eta_1 = 0 \tag{9}$$

Consequently, from (9) and (7), s_0 is an invariant zero of the triple $\{A, D, C_a\}$ and the lemma is proved. $\sharp$

The next section develops an observer scheme for the triple $\{A, D, C_a\}$, based only on knowledge of $y = Cx$, which estimates the states in such a way that the state estimation error is asymptotically stable and independent of the unknown input w once a sliding motion is obtained.

3 Step-by-step observer design

The scheme described in this section will be based on a classical observer of the form (3) for the system $\{A, D, C_a\}$. Consequently this requires (in real-time) the outputs that correspond to $C_a x$ from knowledge of only $y = Cx$. The next subsection describes a scheme to provide these signals.

3.1 A sliding mode observer for a triangular observable form

Here a step-by-step sliding mode observer is designed for a system described by the following triangular form:

$$\begin{cases} \dot{\xi}_1 = \xi_2 + b_1^T u \\ \dot{\xi}_2 = \xi_3 + b_2^T u \\ \vdots \\ \dot{\xi}_{l-1} = \xi_l + b_{l-1}^T u \\ \dot{\xi}_l = b_{l+1}^T \theta + b_l^T u \end{cases},$$
$$y = \xi_1 \tag{10}$$

where $\xi = \begin{bmatrix} \xi_1 & \cdots & \xi_l \end{bmatrix}^T \in \mathbb{R}^l$, $(l > 1)$ is the state vector, $y \in \mathbb{R}$ is the output, $u \in \mathbb{R}^q$ is the known input vector and $\theta \in \mathbb{R}^m$ stands for some unknown inputs. The $b_i^{'} s$ $(i = 1, ..., l+1)$ are vectors of appropriate dimension.

Assume that the system (10) is Bounded Input Bounded State (BIBS) and θ and its first time derivative are bounded, i.e.:

$$\begin{aligned} |\xi_i| &< d_i, \ i = 1, ..., l \\ \|\theta\| &< K \\ \left\|\dot{\theta}\right\| &< K^{'}, \end{aligned}$$

where d_i, K and $K^{'}$ are some known positive scalars.

Most of the sliding mode observer designs for (10) are based on a step-by-step procedure using successive filtered values of the so-called equivalent output injections obtained from recursive first order sliding mode observers (see e.g. [1, 8, 9, 19, 26, 36]). However, the approximation of the equivalent injections by low pass filters at each step will typically introduce some delays that lead to inaccurate estimates or to instability for high order systems. To overcome this problem, this paper proposes to replace the discontinuous first order sliding mode output injection by a continuous second order sliding mode one. The observer is built as follows:

$$\begin{cases} \frac{d\hat{\xi}_1}{dt} = \nu\left(y - \hat{\xi}_1\right) + b_1^T u \\ \frac{d\hat{\xi}_2}{dt} = E_1 \nu\left(\tilde{\xi}_2 - \hat{\xi}_2\right) + b_2^T u \\ \vdots \\ \frac{d\hat{\xi}_{l-1}}{dt} = E_{l-2} \nu\left(\tilde{\xi}_{l-1} - \hat{\xi}_{l-1}\right) + b_{l-1}^T u \\ \frac{d\hat{\xi}_l}{dt} = E_{l-1} \nu\left(\tilde{\xi}_l - \hat{\xi}_l\right) + b_l^T u \end{cases}, \tag{11}$$

where $\tilde{\xi}_1 := y$ and

$$\tilde{\xi}_j := \nu\left(\tilde{\xi}_{j-1} - \hat{\xi}_{j-1}\right), \ 2 \leq j \leq l$$

where the continuous output error injection $\nu(\cdot)$ is given by the so-called super twisting algorithm [24]:

$$\left\{\begin{array}{l} \quad \nu(s)=\varphi(s)+\lambda_s\left|s\right|^{\frac{1}{2}}\operatorname{sign}(s) \\ \quad \dot{\varphi}(s)=\alpha_s\operatorname{sign}(s) \\ \lambda_s,\alpha_s>0 \end{array}\right. . \tag{12}$$

For $i=1,...,l-1$, the scalar functions E_i are defined as

$$E_i=1 \text{ if } \left|\tilde{\xi}_j-\hat{\xi}_j\right|\leq\varepsilon,\ \text{ for all } j\leq i \text{ else } E_i=0$$

where ε is a small positive constant. This is an anti-peaking structure [30]. As argued in [1], with this particular function, the manifolds are reached one by one. At each step, a sub-dynamic of dimension one is obtained and consequently no peaking phenomena appear. Denoting $\bar{\xi}=\xi-\hat{\xi}$, the error dynamics are given by:

$$\left\{\begin{array}{l} \dot{\bar{\xi}}_1=\xi_2-\nu\left(y-\hat{\xi}_1\right) \\ \dot{\bar{\xi}}_2=\xi_3-E_1\nu\left(\tilde{\xi}_2-\hat{\xi}_2\right) \\ \vdots \\ \dot{\bar{\xi}}_{l-1}=\xi_l-E_{l-2}\nu\left(\tilde{\xi}_{l-1}-\hat{\xi}_{l-1}\right) \\ \dot{\bar{\xi}}_l=b_{l+1}^T\theta-E_{l-1}\nu\left(\tilde{\xi}_l-\hat{\xi}_l\right) \end{array}\right. \tag{13}$$

It can be shown (see [18] and [28]) that with a suitable choice of gains λ_s and α_s, a sliding mode appears in finite time on the manifold $\bar{\xi}_1=\cdots=\bar{\xi}_l=0$, and that the following equivalent output injection is obtained:

$$\nu\left(\bar{\xi}_l\right)=b_{l+1}^T\theta$$

Note that the step-by-step observer achieves finite time recovery of the state components.

3.2 First/second order sliding mode unknown input observer

In order to estimate the state of the system (1-2), the following sliding mode observer is proposed:

$$\dot{z}=Az+Bu+G_l\left(y_a-C_az\right)+G_nv_c(y_a-C_az) \tag{14}$$

where the auxiliary output y_a is defined by

$$y_a = \begin{bmatrix} y_1 \\ \nu\left(y_1 - y_1^1\right) \\ \vdots \\ \nu\left(\tilde{y}_1^{\mu_{\alpha_1}-1} - y_1^{\mu_{\alpha_1}-1}\right) \\ \vdots \\ y_p \\ \vdots \\ \nu\left(\tilde{y}_p^{\mu_{\alpha_p}-1} - y_p^{\mu_{\alpha_p}-1}\right) \end{bmatrix} \tag{15}$$

and the constituent signals in (15) are given from the step-by-step observer:

$$\begin{cases} \dot{y}_i^1 = \nu\left(y_i - y_i^1\right) + C_i Bu \\ \dot{y}_i^2 = E_1\nu\left(\tilde{y}_i^2 - y_i^2\right) + C_i ABu \\ \vdots \\ \dot{y}_i^{\mu_{\alpha_i}-1} = E_{\mu_{\alpha_i}-2}\nu\left(\tilde{y}_i^{\mu_{\alpha_i}-1} - y_i^{\mu_{\alpha_i}-1}\right) + C_i A^{\mu_{\alpha_i}-2}Bu \end{cases}$$

for $1 \leq i \leq p$, with

$$\begin{aligned} \tilde{y}_i^1 &:= y_i \\ \tilde{y}_i^j &:= \nu\left(\tilde{y}_i^{j-1} - y_i^{j-1}\right),\ 2 \leq j \leq \mu_{\alpha_i} - 1 \end{aligned}$$

where the injection operator $\nu(\cdot)$ is defined by (12). The discontinuous output injection v_c from (14) is defined by:

$$v_c(y_a - C_a z) = \begin{cases} -\rho\frac{P_2(y_a - C_a z)}{\|P_2(y_a - C_a z)\|} & \text{if } (y_a - C_a z) \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where ρ is a positive constant larger than the upper bound of w. The definition of the symmetric positive definite matrix P_2 can be found in [11] or in Chapter 6 of [12].

If the state estimation error $e = x - z$ and the augmented output estimation error $e_y = C_a x - \bar{y}$, with

$$\begin{aligned} e_y &\triangleq \left[e_1^1, ..., e_1^{\mu_{\alpha_1}-1}, ..., e_p^1, ..., e_p^{\mu_{\alpha_p}-1}\right]^T \\ \bar{y} &= \left[y_1^1, ..., y_1^{\mu_{\alpha_1}-1}, ..., y_p^1, ..., y_p^{\mu_{\alpha_p}-1}\right]^T \end{aligned}$$

then it is straightforward to show that:

$$\dot{e} = Ae + Dw - G_l\left(y_a - C_a z\right) - G_n v_c\left(y_a - C_a z\right) \tag{17}$$

and

$$\begin{cases} \dot{e}_i^1 = C_{_i} Ax - \nu \left(y_i - y_i^1\right) \\ \dot{e}_i^2 = C_{_i} A^2 x - E_1 \nu \left(\tilde{y}_i^2 - y_i^2\right) \\ \vdots \\ \dot{e}_i^{\mu_{\alpha_i}-1} = C_{_i} A^{\mu_{\alpha_i}-1} x - E_{\mu_{\alpha_i}-2} \nu \left(\tilde{y}_i^{\mu_{\alpha_i}-1} - y_i^{\mu_{\alpha_i}-1}\right) \end{cases}$$

$1 \leq i \leq p$. Thus, choosing suitable output injections ν, as shown in section 3.1, the following relations hold after a finite time T:

$$\begin{array}{l} \nu \left(y_i - y_i^1\right) = C_{_i} Ax \\ \nu \left(\tilde{y}_i^2 - y_i^2\right) = C_{_i} A^2 x \\ \vdots \\ \nu \left(\tilde{y}_i^{\mu_{\alpha_i}-1} - y_i^{\mu_{\alpha_i}-1}\right) = C_{_i} A^{\mu_{\alpha_i}-1} x \end{array}$$

for $1 \leq i \leq p$. This means that $y_a = C_a x$. Thus, for all $t > T$, the error dynamics (17) are given by:

$$\dot{e} = (A - G_l C_a) e + Dw - G_n v_c (C_a e) \tag{18}$$

Since by construction $\mathrm{rank}(C_a D) = \mathrm{rank}(D)$ and by assumption the invariant zeros of the triple (A, D, C_a) lie in the left half plane, the design methodologies given in [11], [12] or [31] can be applied so that $e = 0$ is an asymptotically stable equilibrium point of (18) and the dynamics are independent of w once a sliding motion on the sliding manifold $\{e : s = C_a e = 0\}$ has been attained.

In addition, the method given in this paper enables estimation of the unknown inputs. Define $(v_c)_{eq}$ as the equivalent output error injection required to maintain the sliding motion in (18). During the sliding motion, one can write that

$$\dot{s} = C_a \dot{e} = C_a (A - G_l C_a) e + C_a D w - C_a G_n v_c (C_a e) = 0$$

Since $e \to 0$ and using (18):

$$C_a G_n (v_c)_{eq} \to C_a D w.$$

As $C_a D$ is full rank, an approximation $\hat{w}$ of w can be obtained from $(v_c)_{eq}$ by:

$$\hat{w} = \left((C_a D)^T C_a D\right)^{-1} (C_a D)^T C_a G_n (v_c)_{eq}.$$

4 Winding machine example

The developed methodology is illustrated here for a large scale system. A 9-th order web transport system with winder and unwinder for elastic material can be modelled as [23]:

$$\begin{array}{rcl} \dot{x} & = & Ax + Bu + Dw \\ y & = & Cx \end{array}$$

where $x = \begin{bmatrix} J_1\Omega_1 & T_2 & V_2 & T_3 & V_3 & T_4 & V_4 & T_5 & J_5\Omega_5 \end{bmatrix}^T$, $u = [u_u, u_v, u_w]^T$ and $y = [T_u, V_3, T_w]^T$. The signal $w = \begin{bmatrix} w_1(t) & w_2(t) & w_3(t) \end{bmatrix}^T$ represents the unknown inputs vector. The control inputs are the torque control signals applied to three brushless motors driving the unwinder, the master tractor and the winder respectively. The output measurements are the web tensions at the unwinder and winder, T_u and T_w, respectively, and the web velocity, V_3, measured at the master tractor. The states of the system are the corresponding tensions, T_i, and web velocities V_i at various points across the process. The matrices A, B, C and D are given below:

$$A = \begin{bmatrix}
-\frac{f_1}{J_1} & R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{E_0}{L}\frac{R_1}{J_1} & -\frac{V_0}{L} & \frac{E_0}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{R_2^2}{J_2} & -\frac{f_2}{J_2} & \frac{R_2^2}{J_2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{V_0}{L} & -\frac{E_0}{L} & -\frac{V_0}{L} & \frac{E_0}{L} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{R_3^2}{J_3} & -\frac{f_3}{J_3} & \frac{R_3^2}{J_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{V_0}{L} & -\frac{E_0}{L} & -\frac{V_0}{L} & \frac{E_0}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{R_4^2}{J_4} & -\frac{f_4}{J_4} & \frac{R_4^2}{J_4} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{V_0}{L} & -\frac{E_0}{L} & -\frac{V_0}{L} & \frac{E_0}{L}\frac{R_5}{J_5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_5 & -\frac{f_5}{J_5}
\end{bmatrix}$$

$$B = \begin{bmatrix}
K_u & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & K_t\frac{R_3}{J_3} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & K_w
\end{bmatrix}$$

$$C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix}
0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0
\end{bmatrix}$$

Here it is assumed the unknown input distribution matrix is given by $D = B$. Thus, the bounded signal w_i defines the unknown input contribution in the generic system description (1)-(2) and may represent an actuator fault in such a way that $w_i(t) \neq 0$ when a fault appears and is zero in the fault free case.

In the above matrices, V_0, R_i, J_i and f_i are the linear velocity, the radius, the inertia and the viscous friction coefficient of the i-th roll, L is the web length between the i-th and $(i+1)$-th rolls, K_u, K_t and K_w are the torque constants of the three motors. V_0 and E_0 are the nominal values of the linear web velocity and the elastic modulus of the material respectively. The nominal data values used to construct a linear model at start-up are taken from [23] and reported in Table 1.

Notation	Value	Units	Notation	Value	Units
L	0.45	m	J_2	0.00109	kg.m^2
V_0	100/60	m.s^{-1}	J_3	0.00184	kg.m^2
E_0	4175	N.m	J_4	0.00109	kg.m^2
R_1	0.031	m	J_5	0.00109	kg.m^2
R_2	0.02	m	f_1	0.0195	N.m.s.rad^{-1}
R_3	0.035	m	f_2	0.000137	N.m.s.rad^{-1}
R_4	0.02	m	f_3	0.0075	N.m.s.rad^{-1}
R_5	0.032	m	f_4	0.000466	N.m.s.rad^{-1}
J_1	0.0083	kg.m^2	f_5	0.0045	N.m.s.rad^{-1}

Table 1: Parameters of the winding machine

This gives:

$$A = \begin{bmatrix} -2.35 & 0.031 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -34651.94 & -3.7 & 9277.78 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.37 & -0.12 & 0.37 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.7 & -9277.78 & -3.7 & 9277.78 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.66 & -4.08 & 0.66 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.7 & -9277.78 & -3.7 & 9277.78 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.52 & -0.6 & 0.52 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.7 & -9277.78 & -3.7 & 247407.4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.032 & -3.75 \end{bmatrix}$$

The torque constants K_u, K_t and K_w are all set to 1. Note that the triple (A, D, C) has four stable invariant zeros located at $-3.76 \pm 82.43i$ and $-2.15 \pm 97.85i$. Since $CD = 0$, standard UIO approaches cannot be applied to this system. However, the procedure proposed in this paper is applicable. One can choose $\mu_{\alpha_1} = 2$, $\mu_{\alpha_2} = 1$ and $\mu_{\alpha_3} = 2$. Then

$$C_a = \begin{bmatrix} C_1 \\ C_1 A \\ C_2 \\ C_3 \\ C_3 A \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ -\frac{E_0}{2L}\frac{R_1}{J_1} & 0 & 0 & -\frac{V_0}{2L} & \frac{E_0}{2L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{V_0}{2L} & -\frac{E_0}{2L} & 0 & 0 & -\frac{V_0}{2L} & \frac{E_0}{2L}\frac{R_5}{J_5} \end{bmatrix}$$

It can be easily checked that $\mathrm{rank}(C_a D) = \mathrm{rank}(D)$. Consequently using the ideas in §3, the following 'classical' sliding mode observer can be designed:

$$\dot{z} = Az + Bu + G_l\,(y_a - C_a z) + G_n v_c(y_a - C_a z)$$

where v_c is the discontinuous (unit vector) output injection term as in (16) and

$$y_a = \begin{bmatrix} y_1 \\ \nu\left(y_1 - \hat{y}_1\right) \\ y_2 \\ y_3 \\ \nu\left(y_3 - \hat{y}_3\right) \end{bmatrix}.$$

The second and fifth outputs in y_a are produced from (in this case) the degenerate step-by-step observers

$$\begin{aligned} \dot{\hat{y}}_1 &= \nu\left(y_1 - \hat{y}_1\right) \\ \dot{\hat{y}}_3 &= \nu\left(y_3 - \hat{y}_3\right) \end{aligned}$$

where ν is defined by (12).

Define the observation errors as $e = x - z$ and $e_{y_1} = y_1 - \hat{y}_1$, $e_{y_3} = y_3 - \hat{y}_3$. Then the error dynamics are given by:

$$\dot{e} = Ae + Dw - G_l\left(y_a - C_a z\right) - G_n v_c\left(y_a - C_a z\right) \tag{19}$$

$$\begin{aligned} \dot{e}_{y_1} &= C_1\left(Ax + Bu + Dw\right) - \nu\left(e_{y_1}\right) = C_1 Ax - \nu\left(e_{y_1}\right) \\ \dot{e}_{y_3} &= C_3\left(Ax + Bu + Dw\right) - \nu\left(e_{y_3}\right) = C_3 Ax - \nu\left(e_{y_3}\right) \end{aligned} \tag{20}$$

As in [25], choose λ_s and α_s large enough such that after a finite time T_i, $e_{y_i} = \dot{e}_{y_i} = 0$, $i = 1, 3$. This implies that

$$\begin{aligned} \nu\left(e_{y_1}\right) &= C_1 Ax \\ \nu\left(e_{y_3}\right) &= C_3 Ax \end{aligned}$$

and for $t > \max\{T_1, T_3\}$, system (19)-(20) becomes:

$$\begin{aligned} \dot{e} &= \left(A - G_l C_a\right) e + Dw - G_n v_c\left(C_a e\right) \\ \dot{e}_{y_1} &= 0 \\ \dot{e}_{y_3} &= 0 \end{aligned}$$

In the simulations, the following observer parameters have been chosen. The two scalar gains associated with the observers to estimate $\dot{y}_1$ and $\dot{y}_3$ are $\lambda_s = 300$ and $\alpha_s = 8000$. The scalar gain associated with the first order sliding mode discontinuous injection v_c is $\rho = 1.5$. The two matrix gains associated with the linear output error injection feedback and the nonlinear output error injection feedback are:

$$G_l = \begin{bmatrix} 0.029 & -0.00079 & 3.922 & 0 & 0 \\ 15 & 2 & -9277.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 15 & 0 & 9277.7 & 0 & 0 \\ 0.665 & 0 & 12.92 & 0.665 & 0 \\ 3.7 & 0 & -9277.7 & 8.296 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -3.7 & 0 & -9277.7 & 15.7 & 2 \\ 0.00017 & 0 & 0.5 & 0.032 & 0 \end{bmatrix}$$

and

$$G_n = \begin{bmatrix} 13.23 & 7.14 & 33144 & 0 & 0 \\ 123790 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 123790 & 0 & 0 & 0 & 0 \\ 0 & 0 & 123790 & 0 & 0 \\ 0 & 0 & 0 & 123790 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 123790 & 0 \\ 1.85 & 0 & 4642 & 1.85 & 1 \end{bmatrix}$$

respectively. Whilst the scaling matrix in the unit vector injection term is

$$P_2 = \begin{bmatrix} 0.0333 & 0 & 0 & 0 & 0 \\ 0 & 0.0312 & 0 & 0 & 0 \\ 0 & 0 & 0.0294 & 0 & 0 \\ 0 & 0 & 0 & 0.0417 & 0 \\ 0 & 0 & 0 & 0 & 0.0385 \end{bmatrix}$$

For the purpose of demonstration, the control signal u has been set to zero without loss of generality. The unknown inputs have been chosen as follows: w_1 is a square wave of amplitude 0.1 and frequency $0.1Hz$ that starts at $t = 5s$; w_2 is a sine wave of amplitude 0.2 and frequency $1Hz$ that starts at $t = 0s$; w_3 is a sawtooth signal of amplitude 0.05 and frequency $0.4Hz$ that starts at $t = 0s$.

The Figures 1, 2 are related to a test simulation with accurately known parameters in the matrix A. They show that the state is accurately estimated in spite of the three actuator faults. It can be seen in Figure 3 that the unknown input signals are also accurately reconstructed by the proposed scheme.

Robustness tests with respect to parameter variations:

A simulation has been made with a 10% variation of the viscous coefficient f_2. Again, all states were recovered as well as the three unknown inputs. This is shown in Figure 4.

Another simulation for testing robustness issue has been realized by considering a 20% variation of Young modulus E_0. The results of the unknown input reconstruction are shown in Figure 5. The numerical results indicate that the actuator fault detection scheme is tractable even with parameter uncertainties. This is important for instance if several materials with different Young modulus have to be used on the same winding machine.

5 Concluding remarks

In this paper, a new approach to solve the problem of designing a sliding mode unknown input observer for linear systems has been developed. The proposed scheme eliminates the relative degree condition that is inherent in most existing work on unknown input observers. The scheme is based on a 'classical' sliding mode observer used in conjunction with a scheme to estimate a certain number of derivatives of the outputs. The number of derivatives required is system dependent and can be easily calculated. By using the equivalent output injections from the derivative estimation scheme and the classical observer, estimation of both the system state and the unknown inputs can be obtained. Since the derivative estimation observer is based on second order sliding mode algorithms, the equivalent output injections are obtained in a continuous way without the use of low pass filters.

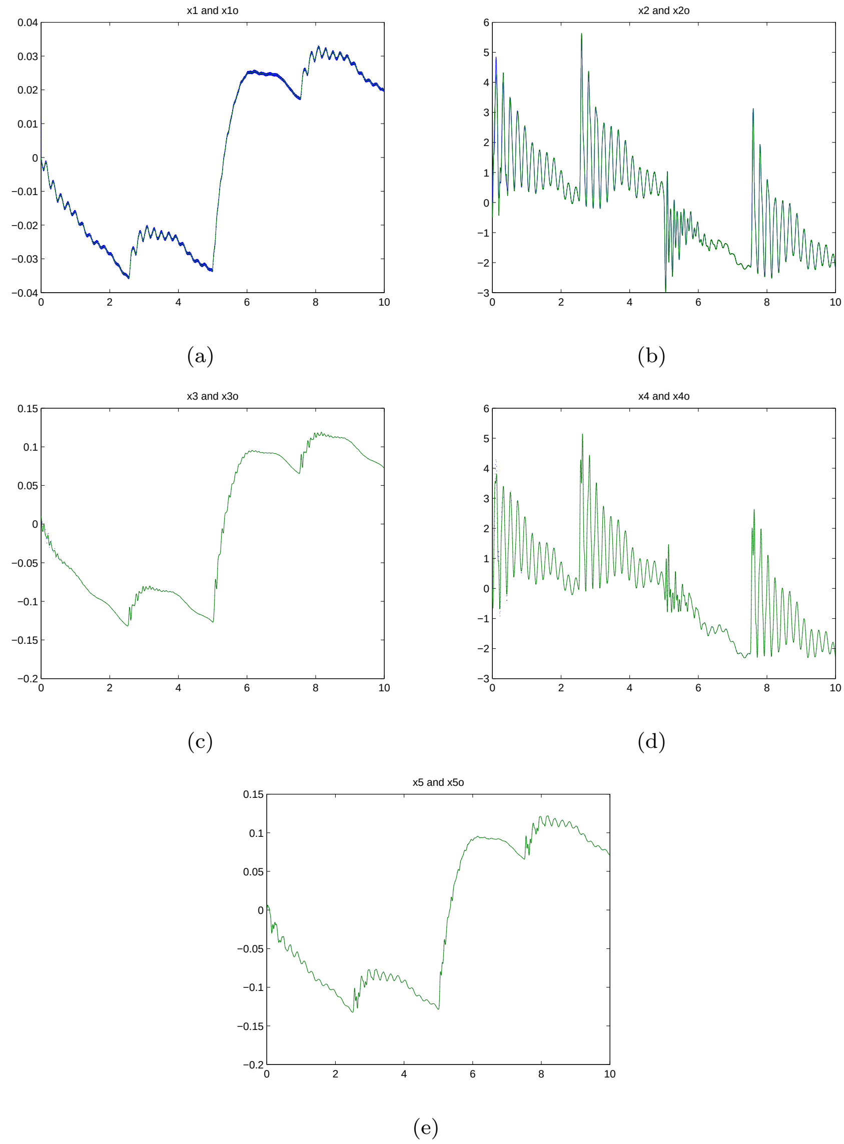

Figure 1: State and estimation (nominal case)

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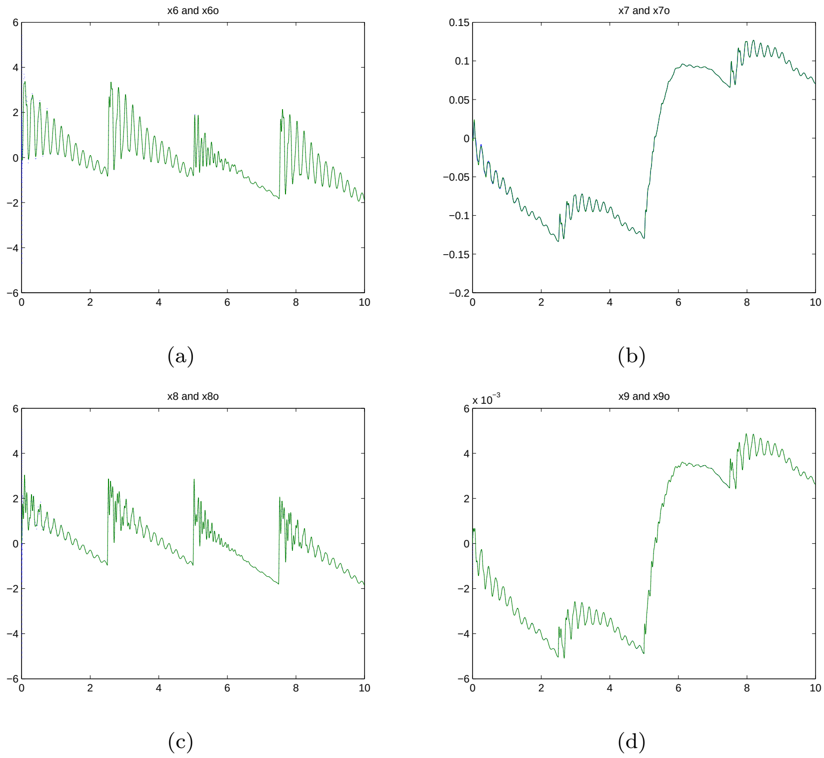

(a) (b) (c) (d)

Figure 2: State and estimation (nominal case)

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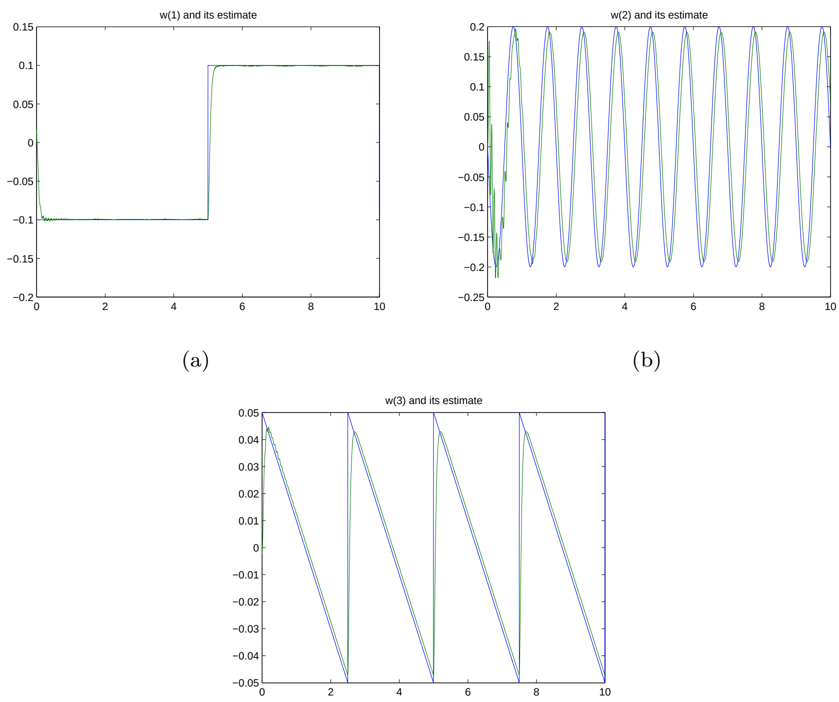

(a) (b) (c)

Figure 3: Unknown input and estimation (nominal case)

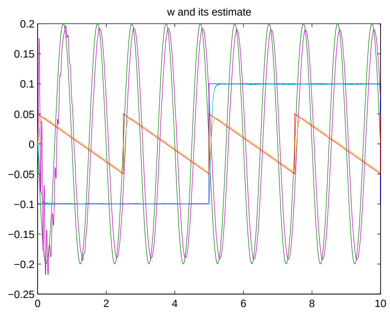

Figure 4: Unknown input and estimation: 10% variation of the viscous coefficient f_2

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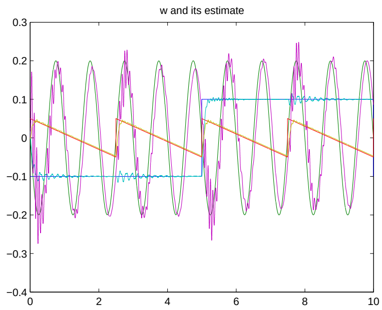

Figure 5: Unknown input and estimation: 20% variation of Young modulus E_0

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